WAVE TRANSMISSION THROUGH AN ASSEMBLY OF RANDOMLY BRANCHING ELASTIC TUBES

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ABSTRACT Calculations are presented of the transmission of oscillations through an assembly of randomly branching elastic tubes, as a model of not only the major arteries, but also a peripheral vascular bed. It appears that the viscosity of the arterial wall must be the major source of attenuation in the larger arteries, while the viscosity of the blood plays a significant role only in the smaller vessels. In all situations, variations of cross-sectional area have a considerable effect on wave transmission, causing a general decrease in amplitude and an accentuation of reflection from the terminations. The effects of variation in cross-sectional area are sufficiently great to indicate that they should be included in future models of the arterial system. Finally, it is argued that because of the presence of random branching and elastic nonuniformity, the determination of the reflection coefficient for a system such as the arterial tree may be quite misleading.

INTRODUCTION

In a previous paper [Taylor, 1966, which will be referred to hereafter as (I)], computations were presented of the input impedance of a branching assembly as a model of the arterial system, to show the manner in which the properties and architecture of the system influenced its input characteristics. It was shown there that the three most important factors were the elastic nonuniformity, the presence of scattered terminations, and the reflections from these terminations. The effect of the viscosity of the fluid or of the wall material was relatively slight, as was the effect of changing cross-sectional area at branching. The present work extends these calculations to show the manner in which the factors listed above determine the propagation of oscillations through the system.

We shall be concerned here mostly with the travel of oscillations in a model of the major arteries, where it will be shown that the important factors are the nonuniform elasticity, reflections from the terminations, viscosity of the wall material, and changing cross-sectional area at branches; fluid viscosity can be expected to have only a slight influence on wave travel in the major arteries. Calculations have, however, also been made on a model "peripheral vascular bed" where it has been found, as might be anticipated, that the effect of the fluid viscosity is much greater and can contribute at least as much to the damping of traveling disturbances as does the viscosity of the wall material.

COMPUTATION

We carried out the computations using the program described in the previous paper (I), to which reference should be made for details. The following modification was incorporated for finding the transmission over any segment or sequence of segments in the assembly. As in (I), a branching of the assembly has been treated purely as the junction of three lines; no account has been taken of the details of fluid motion associated with a bifurcation.

For any uniform segment of elastic tube of length l and propagation constant λ, with reflection coefficient R at the termination, the ratio of pressure at the termination $P(l)$ to pressure at the origin $P(o)$ is given by

$$P(l)/P(o) = (1 + R)/(e^{\lambda l} + Re^{-\lambda l})$$

For any given frequency, this quantity was computed and stored in complex form (as modulus and phase), first for all the terminal segments of the assembly, and then working backward to the origin. When the whole assembly had been covered, it was possible to find the transmission along any pathway through it by multiplying the appropriate transmission factors and adding the phase angles.

The assembly used was the same as that for most of the previous computations (I) except that the first segment has been shortened to bring it somewhat closer to the pattern of arterial lengths found in mammals. We have thus an asymmetrical arrangement, with one set of terminations being considerably closer to the origin than the other. This is analogous to the terminations of the arterial system, which are asymmetrically distributed to the head and upper limbs, and to the more distant viscera and hind limbs.

Two pathways have been chosen through the assembly, one "short" and one "long." These are shown in Fig. 1, with their segments labeled 1 to 8 in each case; these numbers will be referred to in following illustrations.

The program was so constructed that all the parameters of interest could be varied:

1. Terminal impedance, a pure resistance identical at all terminations, calculated from the nominal reflection coefficient at the terminations for the case $d = 1$, i.e., no change in total cross-section area.

2. Fluid viscosity, included by way of Womersley's (1957) nondimensional parameter $\alpha = r\,(\omega/\nu)^{1/2}$, where r = vessel radius, ω = circular frequency = $2\pi f$, ν = kinematic viscosity of the fluid. In the calculations, a value of α_0, was specified for the first segment of the tube for $\omega = 1$, and values for other frequencies and in

FIGURE 1 The assembly of branching tubes used in the calculation; lengths of branches to scale, diameters not to scale. The numbers indicate the short (at left and below) and long (at right and above) pathways over which transmission was studied.

other segments calculated on this basis. In the studies on the major arterial system $\alpha_0 = 15$, while in the calculations on the peripheral vascular bed various values of α_0 from 15 to 2.5 have been used.

3. Cross-sectional area ratio (d), being the ratio of the sum of areas after bifurcation to the area of the parent branch; values used were $d = 1.0, 1.1, 1.2, 1.3$.

4. Viscosity of the wall material, introduced by way of the phase angle (Θ) of the complex elastic constant.

5. The approach of Θ to its asymptotic value Θ_0 was approximated by the expression $\Theta = \Theta_0 . (1 - e^{-\gamma\omega})$. Here, as in (I), an appropriate value of $\gamma = 1$ has been taken. For arterial wall Θ_0 has been found (Bergel, 1961) to be approximately 6° to 8° and almost independent of frequency; the values $\Theta_0 = 0°, 2°, 4°, 6°, 8°, 12°$ have been employed in the calculations.

6. Wave velocity (c) in the various segments of the assembly has been specified as in (I) according to the order of branching;

$$c = 3 - 2.(\tfrac{2}{3})^n$$

where $n = 0$ for the first segment. For the model of the peripheral vascular bed $c = 3$ for all n.

7. Distribution of branch lengths. The same assembly (Fig. 1) was used for all computations. The mean lengths of branches of order n were chosen equal to $1/(n + 1)$ and the lengths of branches were distributed about their means according to the second order Γ-distribution (see I), except for the zero-order segment, which was fixed at 0.2 units. In calculations on the model peripheral vascular bed, all lengths were divided by four to keep the assembly in reasonable proportion to the model of the major arterial tree.

8. Among the data upon which the program operated two lists of numbers specifying the two pathways chosen through the assembly were included and referred to as long and short.

At each value of frequency ($\omega = 0 - 4\pi$ by steps of $\pi/20$), the program printed out:

1. Frequency.
2. Input impedance in modulus and phase.
3. Transmission ratio for pressure at each branch (1 to 8) along the two chosen pathways; the pressure amplitude at the origin was always taken to be unity.
4. The progressive phase shift.
5. The apparent phase velocity over each segment along the two chosen pathways.

Computation time was about 1 sec per set of results for one value of ω.

RESULTS

Input Impedance

This will not be discussed in detail, having been dealt with in (I). Fig. 2 is included here, however, to show the input properties of this particular assembly, with a reasonable choice of area ratio ($d = 1.2$) and two values of the wall-viscosity parameter $\Theta_0 = 2°, 8°$. The modulus of the impedance shows two minima close together in the low-frequency range, which can be attributed to the asymmetrical nature of the assembly. This double minimum is very similar to that observed in the input impedance of the mammalian arterial system (O'Rourke and Taylor, 1966 *b*), where the distribution of branches is likewise asymmetrical. As has been discussed previously, the main features of the input impedance depend upon the architecture and elastic properties of the system and, as can be seen from Fig. 2, are only slightly affected by the presence of a viscoelastic wall. The inset curve shows the remarkable stabilization of the input impedance which this system exhibits; for $\omega > \frac{1}{2}\pi$, the impedance modulus shows only slight fluctuations about the value 1.0, which is the characteristic impedance of the first segment of the assembly.

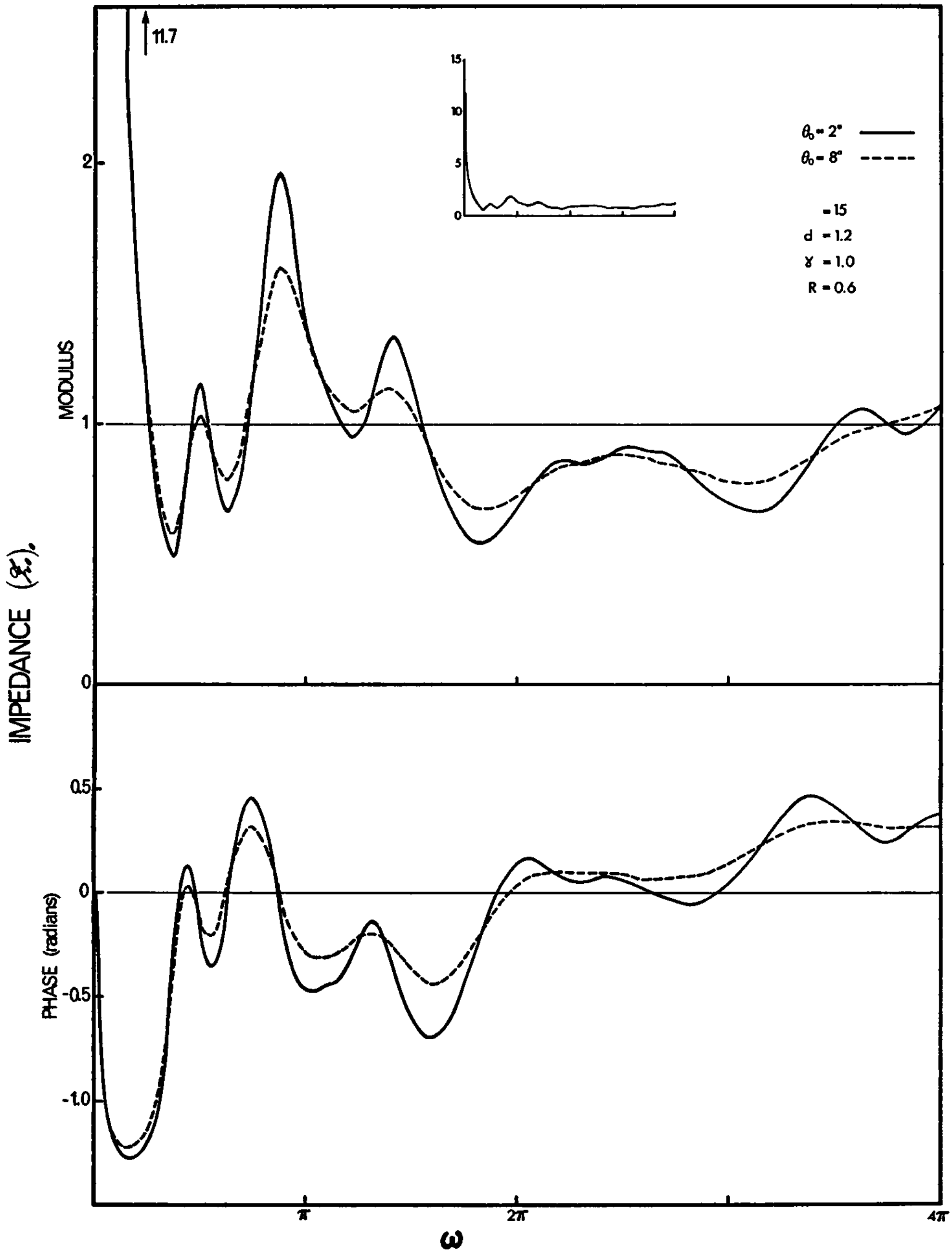


FIGURE 2 Input impedance of the assembly; modulus (above) phase angle (below). The parameters are given in the upper panel, which also includes a small diagram of the whole course of the modulus. Note that the impedance is only slightly changed by a fourfold increase in wall viscosity ($\Theta_0 = 2°$, $\Theta_0 = 8°$).

Pressure Transmission through the Assembly

It has been shown elsewhere (Taylor, 1964, 1965) that in an elastic tube in which wave velocity is increasing with distance from the origin, oscillations will undergo "amplification" as they travel. In those papers, calculations were presented to show the form of this amplification, as a function of frequency, with and without reflections from the termination, and attention was drawn to its role in the transformation of pressure pulses traveling in the arterial system. The model chosen for those calculations was of only a single elastic tube without branches, of constant cross-section, and without viscous elements in either the fluid or the wall. In the following sections of this paper we examine the influence of these various factors on transmission through, first, a model major arterial system, and, secondly, a model peripheral vascular bed.

Area Ratio. The transmission of pressure oscillations was calculated for passage through the assembly by paths shown, to the terminations of the long and short pathways (see Fig. 1, where these are labeled 8). The amplitude at the origin was taken to be unity. Two cases were chosen, one with no viscous element in the wall ($\Theta_0 = 0°$, Fig. 3) and the other with a reasonable value for this ($\Theta_0 = 8°$, Fig. 4). In both cases $\alpha_0 = 15$, $R = 0.6$. The behavior of the transmission ratio is shown for various values of the area ratio d (1.0, 1.1, 1.2, 1.3) at the bifurcations.

It will be noted that the curves do not start at 1.0 for zero frequency; this is due to viscous losses in the fluid, and represents the *dc* pressure drop along the system. It will be seen that this is less when d is large, which would be expected from the decreased pressure drop through a system of increasing cross-sectional area. In the absence of wall-damping and for a system of constant cross-sectional area (Fig. 3, $d = 1.0$, solid line) transmission is similar to that previously calculated for a single nonuniformly elastic tube (Taylor, 1965). The maxima and minima are due to the presence of reflected components, but the fact that the minima do not return to the value unity is due to the presence of elastic nonuniformity, with higher wave velocity in the more distant branches. The gradual decline in transmission at higher frequencies is due to the attenuation by fluid viscosity, but this is slight in comparison with the influence of wall viscosity. When wall-damping is present, (Fig. 4) the amplification and reflection effects are considerably reduced, and at high frequencies the attenuation due to the viscous loss is very marked and the transmission falls below unity.

Increasing the cross-sectional area ratio (larger values of d) accentuates the effects of reflections at low frequencies, but otherwise generally reduces the transmission. For a large area change ($d = 1.3$) such that the total cross-sectional area of the seventh-order branches is more than six times the original value, we find that in the absence of wall-damping the effect of reflection is marked, giving a high initial peak and a deep minimum, with considerable fluctuation thereafter. This is partly because the calculations were carried out, as explained under item 1 of the

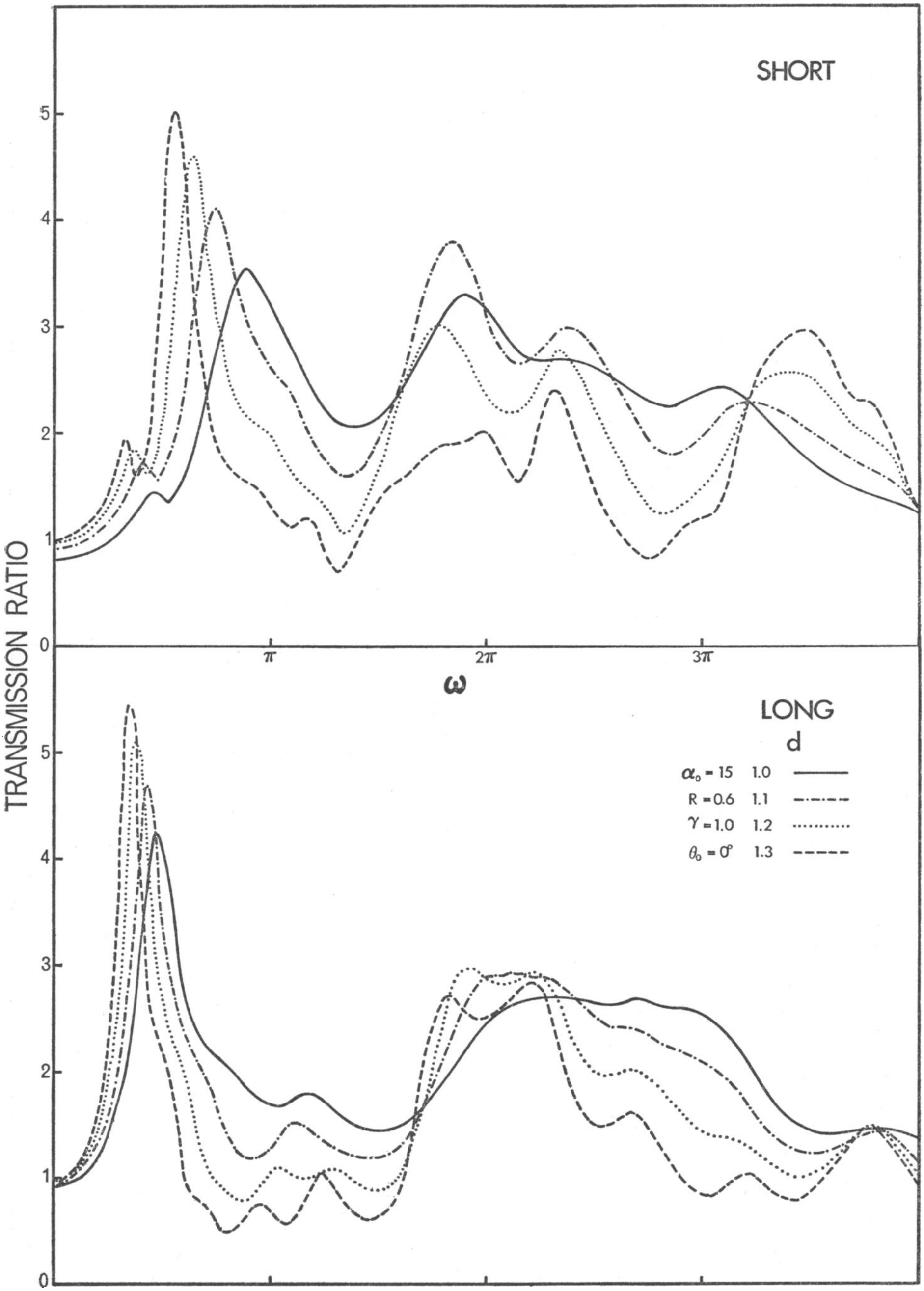


FIGURE 3 Transmission over short (upper panel) and long (lower panel) pathways through the assembly; parameters as shown; fluid viscosity only. Note that with increasing area ratio (d) at the bifurcation there is accentuation of the effects of reflections but otherwise a general decrease in transmission.

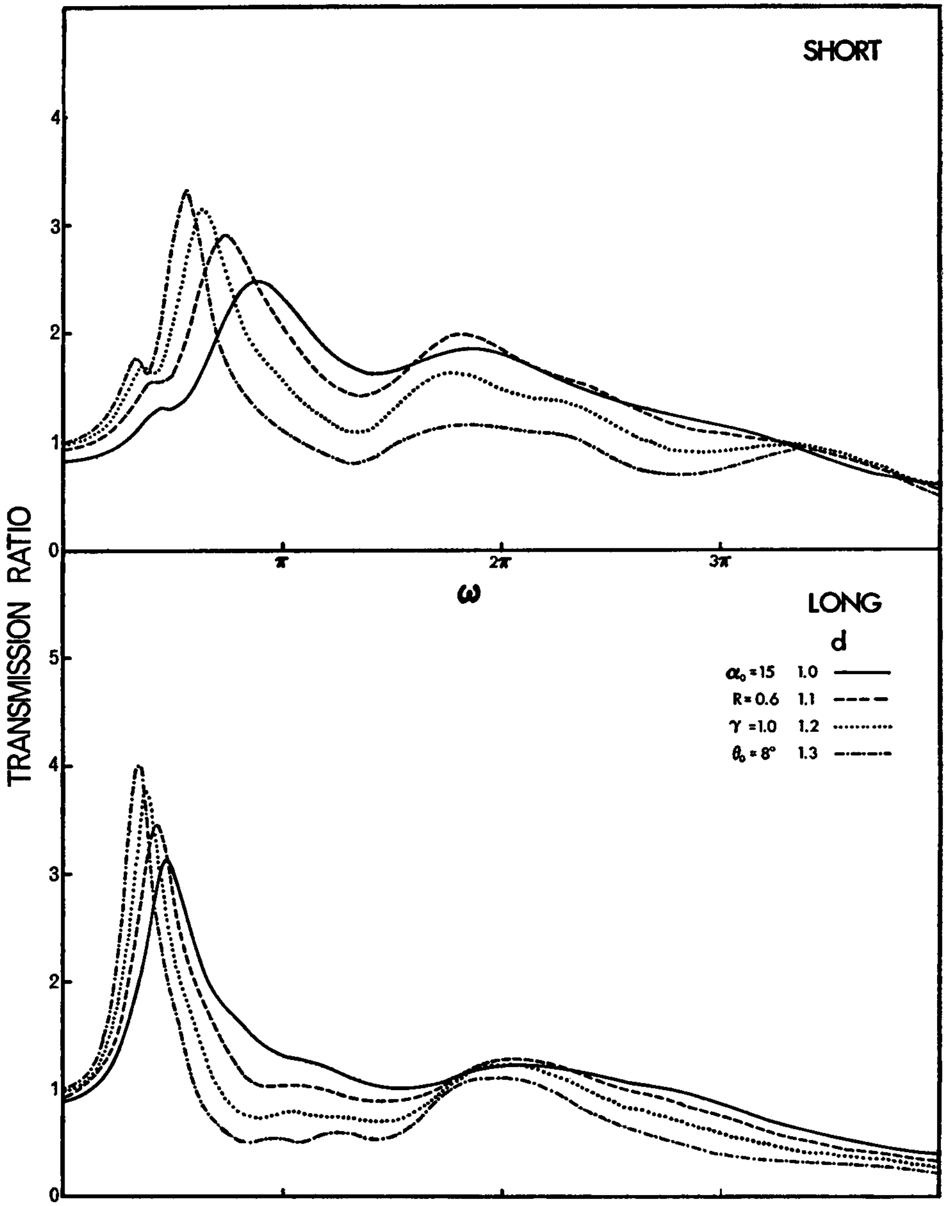


FIGURE 4 As for Fig. 3 except that wall viscosity is now also included ($\Theta_0 = 8°$). Transmission is reduced at higher frequencies and the effects of reflections are also less marked.

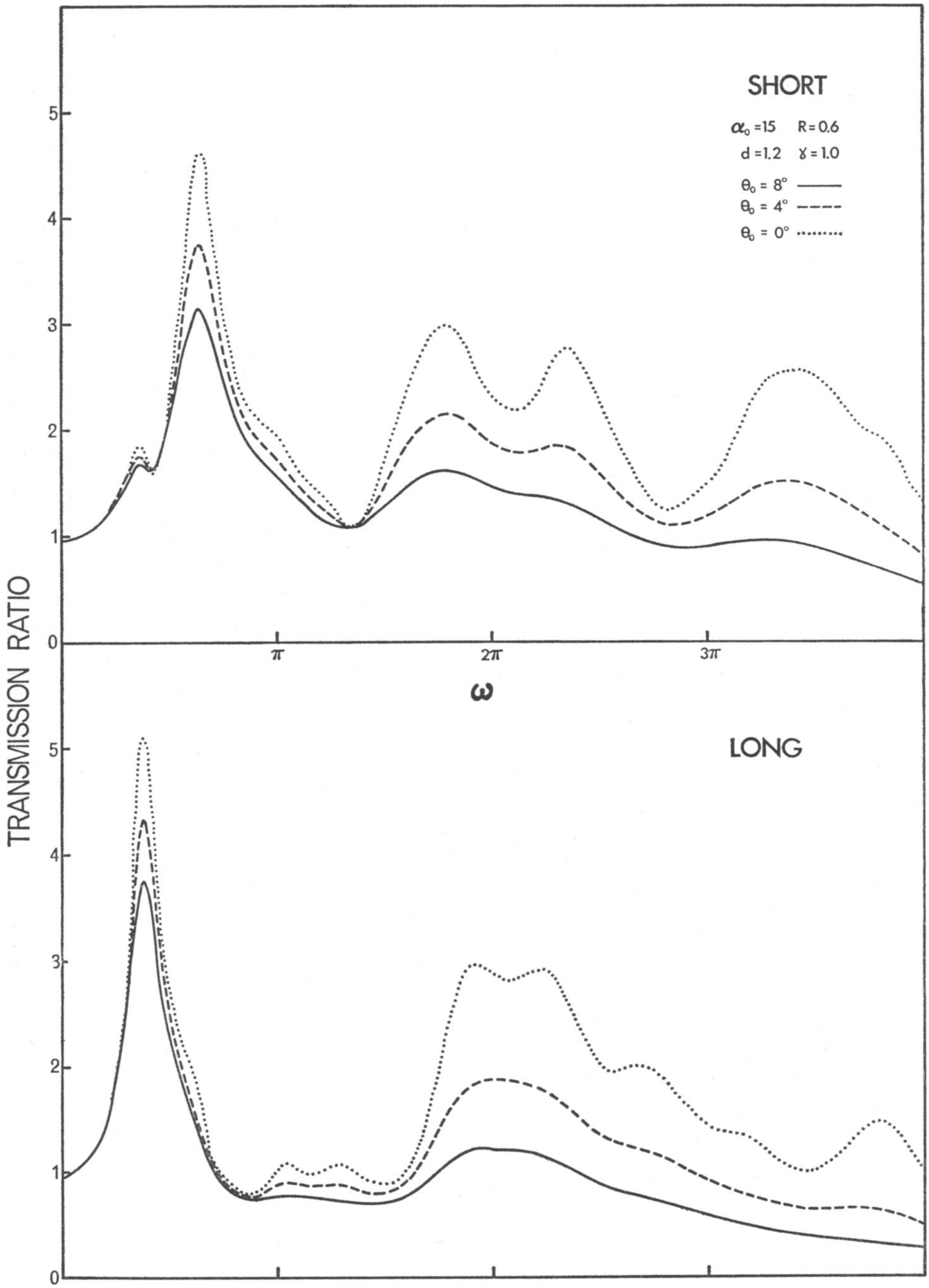


FIGURE 5 As for Figs. 3 and 4, to show the influence of various degrees of wall viscosity ($\theta_0 = 0°, 4°, 8°$) on transmission through the assembly.

computational methods, with a fixed value of the terminal impedance, calculated from the specified reflection coefficient for $d = 1.0$. For $d = 1.0$ the increased cross-sectional area of the terminal branches reduces their characteristic impedance proportionately, so that for $d = 1.3$, the actual reflection coefficient increases from 0.6 to 0.95.

A second cause of the changed behavior with increasing values of d was discussed in (I), and is related to the loss of reflections from the intervening junctions (Womersley, 1957), so that the "effective" termination moves away from the origin; thus the resonant frequency of the system, which gives rise to the first maximum of transmission, is decreased.

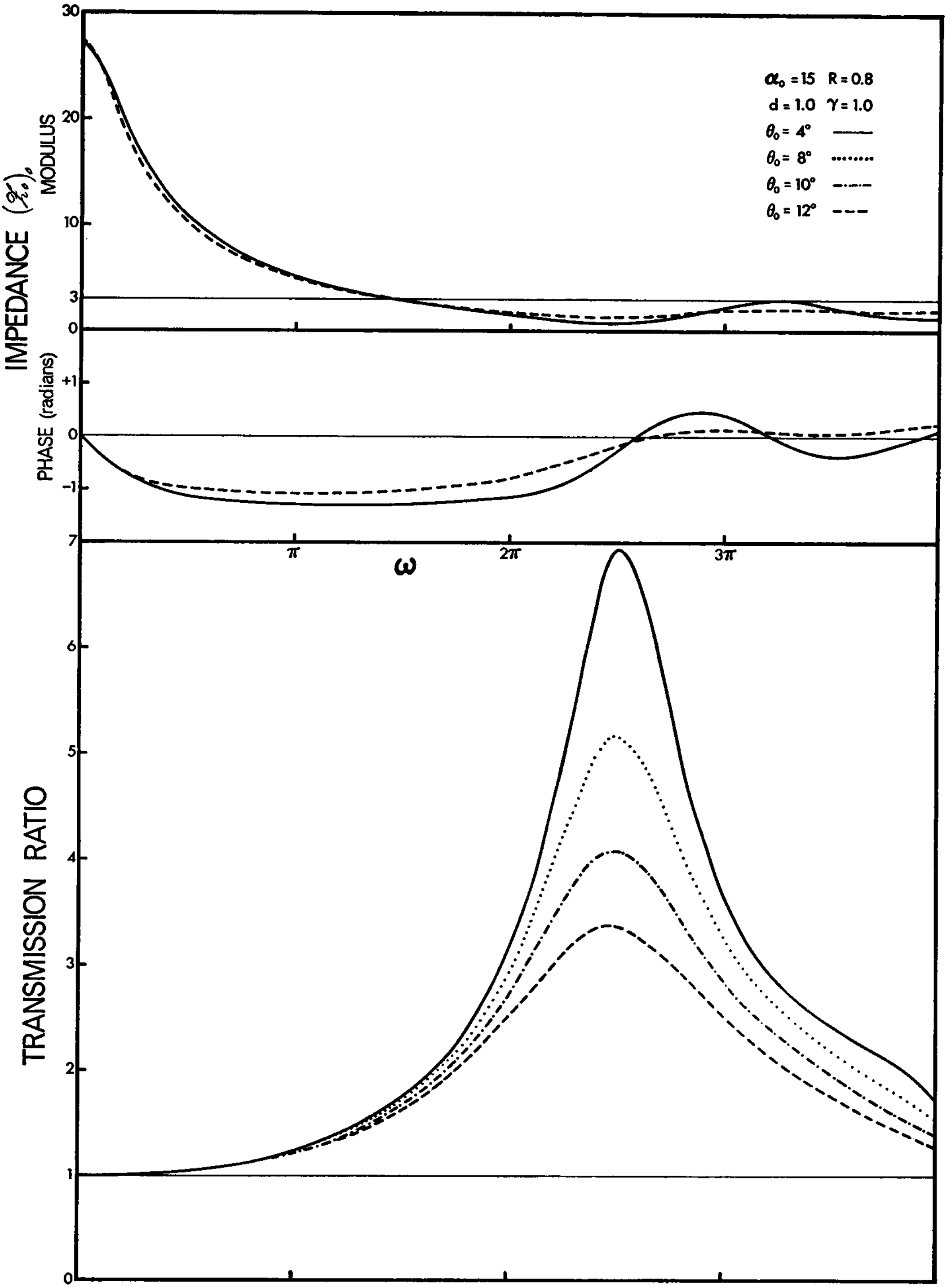
An expansion of the total cross-sectional area of the system results in a general fall in amplitude of oscillations traveling in it, as is clearly seen in these examples. It is particularly clear in Fig. 4, where the effect of reflections is diminished by the viscous losses at higher frequencies.

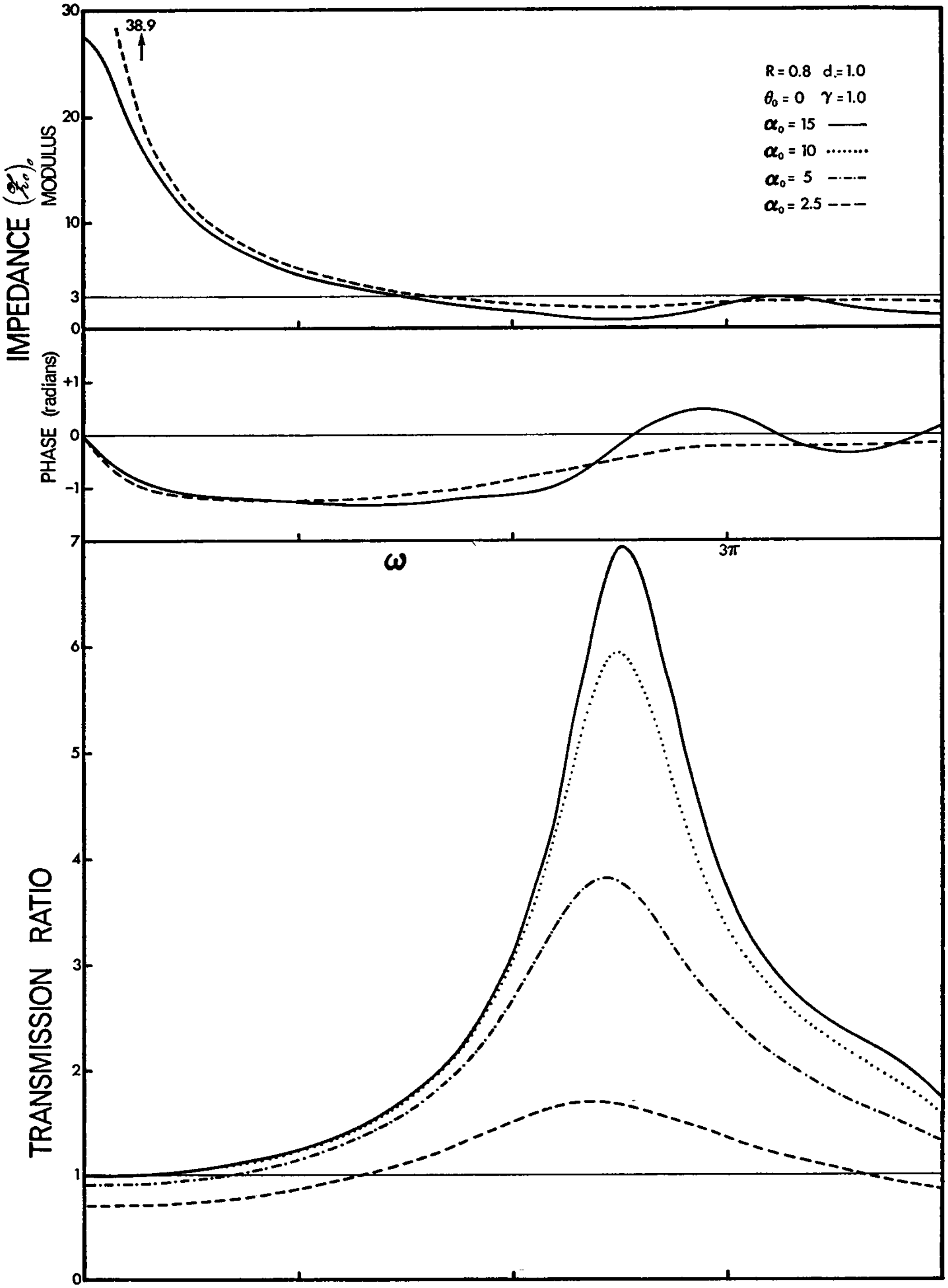
Viscoelastic Wall. The influence of viscous losses in the walls of the tubes can be seen by the comparison of Figs. 3 and 4, but to make this clearer, three cases are shown in Fig. 5. The calculations here were for the parameters $d = 1.2$, $R = 0.6$ (for $d = 1$) with $\alpha_0 = 15$, and $\Theta_0 = 0°, 4°, 8°$. As can be seen, a relatively small phase angle in the dynamic elastic constant has a marked effect on transmission. The amount of wall viscosity present in arteries can thus be expected to provide significant attenuation of traveling waves although the final behavior is, of course, also determined by the other properties of the system.

A Peripheral Vascular Bed. In the previous two sections we have dealt with transmission through a model of the major arteries, and attention has been focused on the influence of wall-viscosity and area changes. For purposes of comparison, the calculations have been extended to a vascular bed where, by reason of the smaller diameters of the vessels, the fluid viscosity might be expected to play a larger role. As was done in the previous paper (I) we have therefore considered an assembly in which the wave velocity is constant in all segments of ($c = 3$ units) and which has been suitably scaled down in length, (see paragraph 7 under Computation).

Fig. 6 shows the transmission ratio for oscillations traveling through the long path of this assembly, where the effects of fluid viscosity have been made very small by choosing the quite unrealistic value of $\alpha_0 = 15$. A high reflection coefficient ($R = 0.8$) was used here, since a value of this order has been found appropriate for regions such as the femoral vascular bed (O'Rourke and Taylor, 1966 *a*). The

FIGURE 6 Input impedance (above) and transmission ratio for the long pathway (below) for a model regional vascular bed, to show the influence of various degrees of wall-damping ($\Theta_0 = 4°$ to $12°$). Fluid viscosity is effectively very small in these examples ($\alpha_0 = 15$).


IMPEDANCE
MODULUS
PHASE (radians)
α_0 = 15 R = 0.8
d = 1.0 γ = 1.0
θ_0 = 4°
θ_0 = 8°
θ_0 = 10°
θ_0 = 12°
30
20
10
3
0
+1
0
-1
7
6
5
4
3
2
1
0
π
2π
3π
ω
TRANSMISSION RATIO



IMPEDANCE $(\mathscr{Z}_{c})_0$ MODULUS
PHASE (radians)
TRANSMISSION RATIO
38.9
R = 0.8 d = 1.0
θ_0 = 0 γ = 1.0
α_0 = 15
α_0 = 10
α_0 = 5
α_0 = 2.5
ω
3π

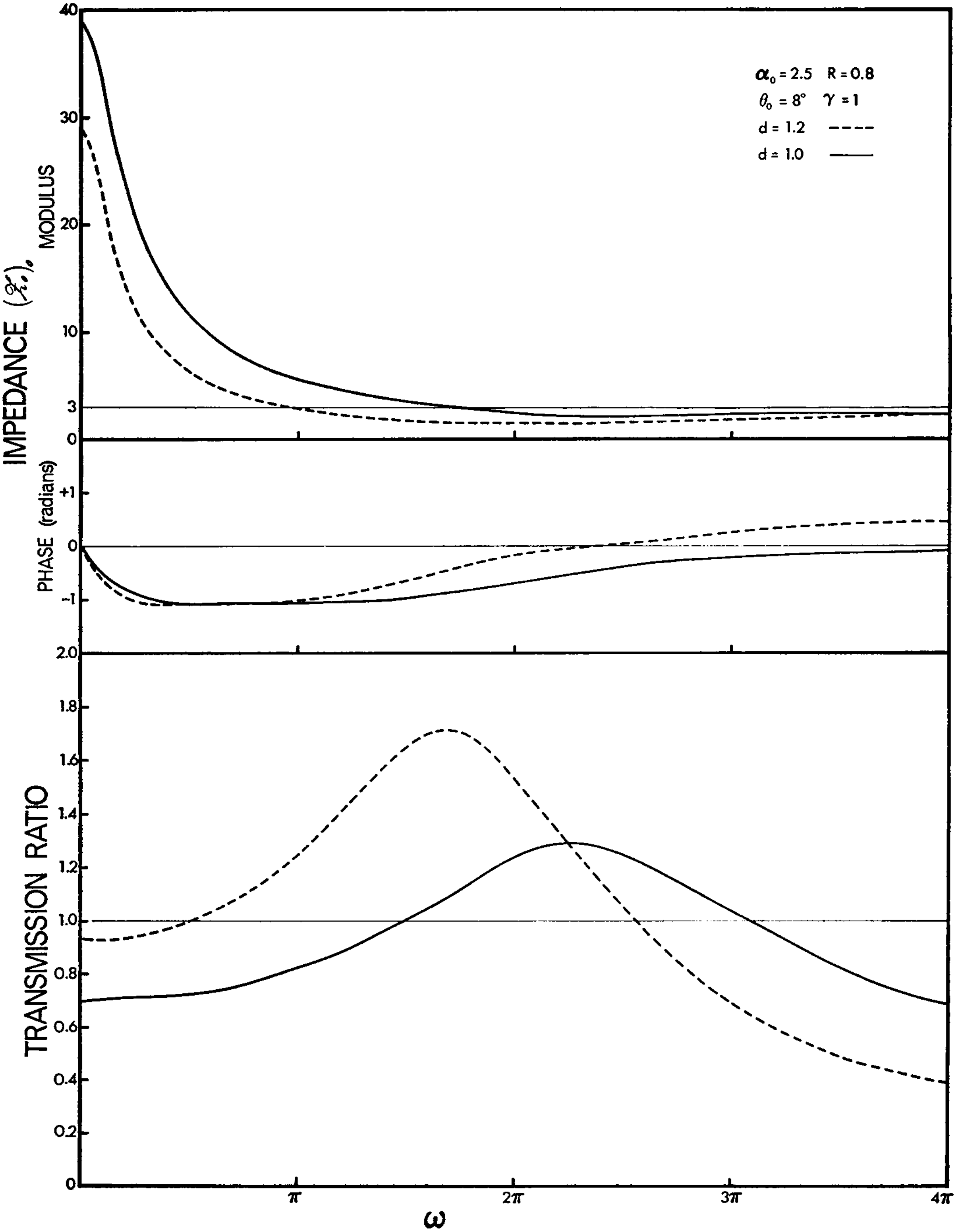
progressive attenuation with increasing values of $\Theta_0 = 0°, 4°, 8°, 12°$ is clearly shown.

Fig. 7 shows the effect of decreasing α_0 in the absence of wall viscosity ($\Theta_0 = 0°$). With values of α_0 appropriate to such a vascular bed ($\alpha = 5, 2.5$), the attenuation which a traveling wave undergoes is of the same order as or greater than that due to the presence of a viscoelastic wall.

The combined influence of these two sources of attenuation is considered in Fig. 8, where two cases are taken, $d = 1.0$ and $d = 1.2$ with $R = 0.8$, $\alpha_0 = 2.5$, $\Theta_0 = 8°$. One sees that in the second case ($d = 1.2$) where all the parameters of the system have reasonable values, such as would be found for a regional arterial bed in an animal, the transmission to the terminal branches increases slowly over the low frequencies to reach only a low maximum, after which it falls off rapidly at the higher frequencies. It should be pointed out that in these cases we are dealing with transmission into quite small vessels. If, for example, we take the radius of the parent branch to be 2 mm then the radius of a terminal branch of the assembly, (for $d = 1.2$) is $2(1.2/2)^{7/2}$ mm = 0.33 mm. This is rather smaller than the vessels in which pressure and flow are usually examined. These three figures (6 to 8) also include plots of input impedance, which are very similar in form to those previously computed (I, Fig. 12). It is clearly shown in Figs. 6 and 7 that while viscous damping may have marked affects on transmission, the input impedance remains relatively unchanged. If fluid viscosity is increased (small α_0) then the *dc* resistance of the system is increased and the impedance rises further at zero frequency (Fig. 7) but otherwise there is little change. The results emphasize once again the dominance of architectural and elastic structure of the system in determining the input impedance.

Transformation of a Pressure Pulse in Travel. One of the striking features of pressure pulses in arteries is the manner in which they undergo changes in shape during their travel along the aorta and into the peripheral arteries. As has been discussed in earlier publications (Taylor, 1964, 1965) these transformations are largely explicable on the basis of reflections and elastic nonuniformity. The opportunity has been taken, however, to calculate the behavior of pressure pulses traveling in the model assembly for the major arteries. The parameters of the model wave $\alpha_0 = 15$, $d = 1.2$, $\Theta_0 = 8°$, $R = 0.6$. The input to the system was a typical aortic flow-pulse in a dog, recorded by an electromagnetic flowmeter, and resolved into its first five Fourier components. The value $\omega_1 = 0.39$ was chosen for the fundamental. In Fig. 9 the transmission ratios are shown for the five pressure

FIGURE 7 Input impedance (above) and transmission ratio for the long pathway (below) for a model regional vascular bed, to show the influence of various degrees of fluid viscosity; a value of $\alpha_0 = 5$ or 2.5 is appropriate to such a region. Wall viscosity is excluded ($\Theta_0 = 0°$).



IMPEDANCE ($\mathcal{Z}_0$), MODULUS
PHASE (radians)
TRANSMISSION RATIO
α_0 = 2.5 R = 0.8
θ_0 = 8° γ = 1
d = 1.2
d = 1.0
40
30
20
10
3
0
+1
0
-1
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
π
2π
3π
4π
ω

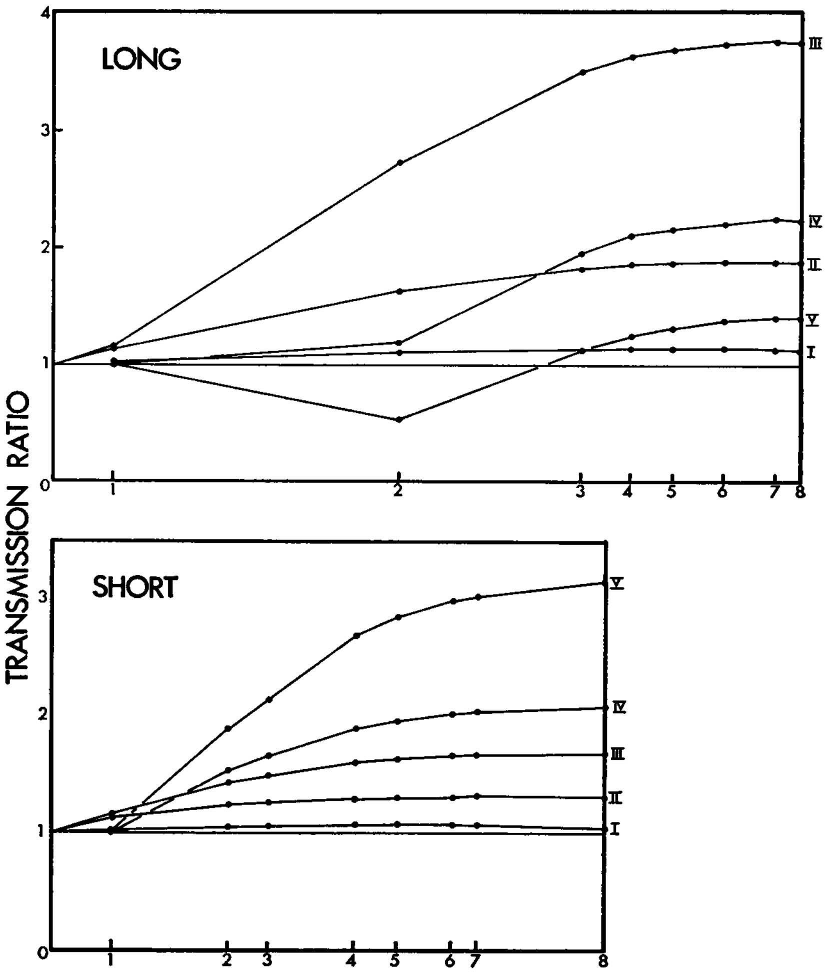


FIGURE 9 Transmission ratios for the first five harmonic components ($\omega_1 = 0.39$) of a pressure wave plotted along the two pathways through the assembly; the parameters are given in the text.

harmonics concerned, plotted at the eight points along the two pathways. There is a close resemblance to the results of similar analysis of pressure pulses in the arterial system. Fig. 10 shows pressure pulses synthesized at several points along the pathways. The "peaking" of the pulse traveling along the long pathway is very

FIGURE 8 Input impedance (above) and transmission ratio for the long pathway (below) for a model "regional" vascular bed. Both fluid viscosity and wall viscosity are present ($\alpha_0 = 2.5$, $\Theta_0 = 8°$), two cases are given for $d = 1$ (constant total cross-section) and $d = 1.2$ (expanding total cross-section).

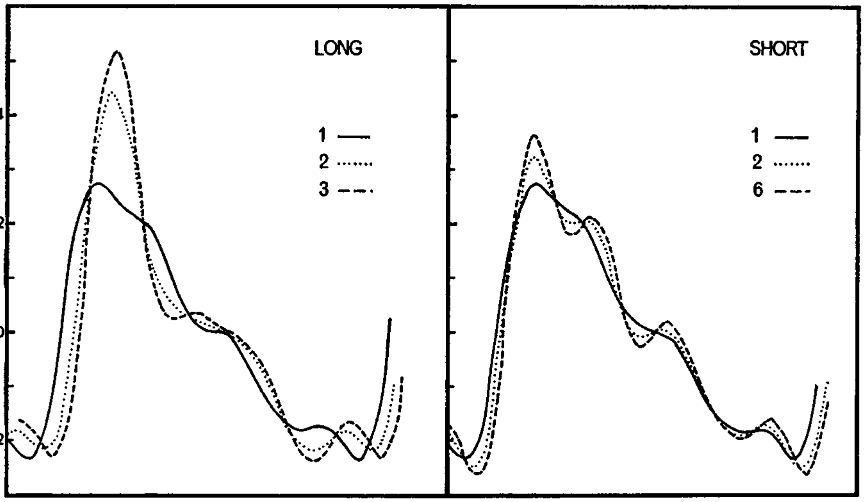


FIGURE 10 Synthesized pressure pulses at various points in the assembly, using as input at the origin a normal cardiac ejection pulse recorded from a dog. The numbers refer to the points in the assembly (see Fig. 1). Note the greater "peaking" of the wave form in its passage through the long pathway.

marked and is greater than that shown by the pulse in the short pathway. These examples agree with the well known differences in form between the pulse waves in the femoral and brachial arteries (Wiggers, 1928,: Kroeker and Wood, 1955).

Apparent Phase Velocity. In the previous paper (I) there was some discussion of the behavior of the apparent phase velocity in various parts of the assembly. It was stated that because of the presence of scattered terminations, one would expect to find the apparent phase velocity in the distant branches to be strongly influenced by reflections, but to be relatively stable in the more central segments.

Fig. 11 shows a selection of results for the present assembly ($R = 0.6$, $d = 1.2$, $\Theta_0 = 8°$, $\alpha_0 = 15$); the ringed numbers denote the termination of the segment over which the apparent phase velocity has been computed (cf. Fig. 1). In each case the true phase velocity for that segment is indicated by a thin line. It is clear that in the "central" regions [(1), short (2) and long (2)], for values of ω greater than about $\pi/4$, the apparent phase velocity becomes relatively stable and almost equal to the nominal value for those regions. In the more peripheral extensions of the assembly [short (5) and long (6)] the deviation of the apparent phase velocity from its true value is much greater, and persists to higher frequencies.

These results are not strictly comparable with those which might be obtained in an animal experiment, because they have been computed from the phase differences over whole segments of the assembly, of different length. Thus some smoothing out may well have occurred in those estimations which are over segments such as long

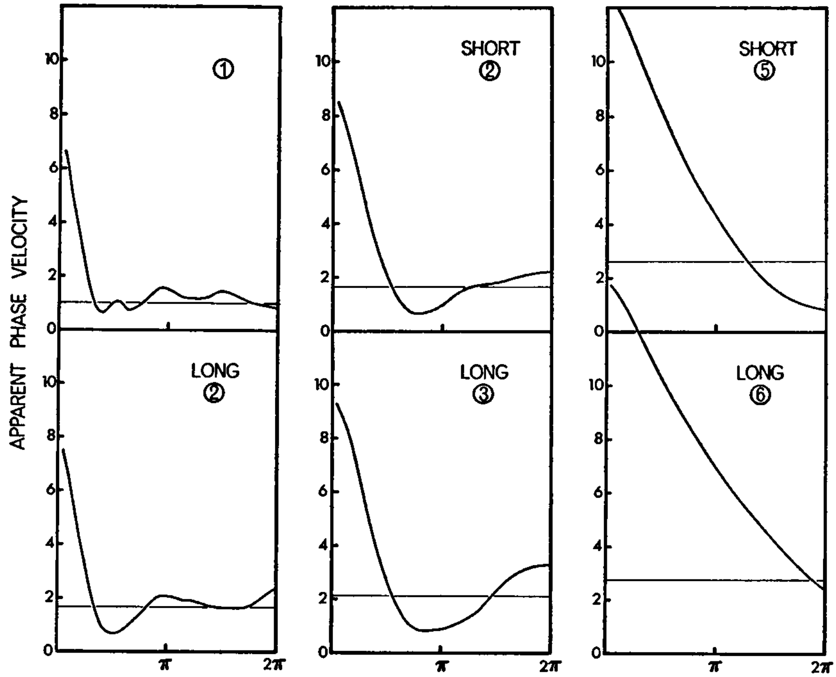


FIGURE 11 Apparent phase velocities over various segments of the assembly; true phase velocities indicated by a thin line. Note the rapid stabilization of apparent phase velocity in the central region. In all instances the abscissa is circular frequency (ω).

(2) and long (3). In an animal experiment it is usual to compute apparent phase velocity from measurements of phase differences made over a small fixed interval (McDonald and Taylor, 1959), but it would not have been very convenient to incorporate such a procedure in the present computational scheme. The results are presented only briefly, but their general interpretation is clear, and they serve as an illustration of the behavior to be expected in systems of this kind.

DISCUSSION

Four main conclusions may be drawn from the results presented in this paper.

1. Although viscous properties have only minor effects upon input impedance, they have marked effects on the behavior of traveling waves.
2. Transmission along larger vessels is mainly influenced by wall viscosity, and only slightly by fluid viscosity; in smaller, "peripheral" vessels the influence of fluid viscosity becomes progressively greater.
3. The presence of scattered terminations and elastic nonuniformity severely interferes with the estimation of the terminal reflection coefficient.
4. The changes in cross-sectional area must always be considered.

On examining the relation between transmission and input impedance, one finds that as the frequency is increased, the input impedance becomes almost constant; this is because the elastic properties are nonuniform and the terminations are randomly scattered. For sufficiently high frequencies the input impedance becomes stabilized and the addition of viscous damping makes very little difference. On the other hand, if we follow an oscillation along any particular pathway from the origin to one of the terminations, we see that its amplitude depends upon the degree to which it is attenuated in travel, and also upon the presence of reflected components. It is clear from the results presented in Fig. 5 that only a small amount of damping was provided by fluid viscosity in the larger vessels (large α). The insertion of an appropriate amount of wall viscosity ($\Theta_0 = 8°$) produced a considerable reduction in the transmission peaks at resonance, and largely abolished the subsequent variations. In a model of the peripheral vessels, however, it was found that fluid viscosity and wall viscosity could play large and approximately equal roles in the attenuation of oscillations. When taken together (Fig. 8) it was found that only small resonance effects could be expected. Some unpublished observations from this laboratory (Fox, 1961) on the transmission of the pulse wave from the aorta to the small mesenteric arteries in the cat, have shown a similar form, the maximum of the transmission ratio being between 1.2 and 1.4 at about 14 cycle/sec. One may therefore conclude that the present calculations are based on a reasonable description of this type of vascular bed, even though the α-values in the mesenteric vessels were smaller than those employed in the present calculations.

When considering the influence of area changes, the previous study (I) showed that if the area ratio at branching was greater than unity, the minimum of the impedance modulus was moved to a lower frequency, but the general behavior of the impedance was otherwise little changed. These mild effects are seen in the upper part of Fig. 8. When the total cross-section was increased with branching, the *dc* resistance of the assembly was reduced, as would be expected, and the shallow minimum moved to the left. The effects on transmission were more pronounced, and the resonant frequency, for the maximum transmission ratio, was decreased. For frequencies above the resonant one (Figs. 3 and 4), the general effect of an expanding cross-section was to reduce the amplitude of the oscillations.

As was discussed under Area Ratio, this dependence on area ratio can be attributed to the changing reflection coefficient at the branching; the "effective reflection site" is shifted further from the origin if the intermediate reflections are reduced by the increased area ratio. This brings up the whole question of what is meant by the term reflection coefficient in this situation. While it can properly be applied to any particular discontinuity in the assembly, such as a branching or a termination, its use in a more general fashion is by no means straightforward. Knowing the characteristic impedance, wave velocity and length of a single, uniform elastic tube, it is possible to estimate the reflection coefficient at its termination by measuring the input impedance or the apparent phase velocity. If the input impedance of an assembly (Fig. 2) were used in this way, assuming the system to be single, with a single termination, the estimated reflection coefficient would approach zero as the frequency was increased. This is made still clearer by Fig. 11. Estimates of the reflection coefficient, based on measurements of the apparent phase velocity made near the origin, would indicate that for $\omega > \frac{1}{2}\pi$ the reflection coefficient was very small; the same measurements, made near the terminations, e.g. long (6), would indicate a much larger value. While for a uniform tube the input impedance or apparent phase velocity depends, at all frequencies, upon the terminal conditions, this is not true for a nonuniform tube or a branching assembly. As the frequency is increased, the terminations and their reflections have less and less influence upon the input impedance, and the analysis in terms of a single-tube analogy must yield misleading results. To imitate the behavior of the input impedance of an assembly such as that given in Fig. 2, or such as the arterial system, by means of a single uniform model, one would have to prescribe either a nonexistent frequency dependence in the terminal impedance, or an equally unrealistic and extreme degree of attenuation of the traveling waves. It is thus clear that considerable caution will have to be exercised in making and interpreting measurements of the reflection coefficient of the arterial system.

The final form of the relationship between transmission and frequency depends upon the interaction of a number of factors. First, if reflections are present, then at certain resonant frequencies the transmission ratio will have maxima and minima. In a single, ideal, uniformly elastic system the minima of transmission will occur whenever there is an antinode of pressure at the origin, and will have the value unity. In a nonuniformly elastic system, as has been shown previously, this minimum value will not be unity, but will tend to the value of the square root of the nominal characteristic impedance of the terminal segment. In the arterial system, because of the increase in wave velocity in the peripheral arteries, we may expect this to lead to "amplification" of a traveling wave. Attenuation due to viscous losses will cause a reduction in transmission, with a less marked resonant peak and a progressive decrease at higher frequencies. However, in a nonuniformly elastic system, if the damping is not too great, the first minimum of the transmission ratio may still be greater than unity. Finally, if we take account of the expanding total cross-sectional area of the system, we find that this not only leads to an accentuation of reflections, with consequently more marked maxima and minima, but also to a general decrease in transmission, which may be sufficient to offset the amplification arising from the elastic nonuniformity. These effects are sufficiently striking to make it clear that allowance for area changes should be incorporated in future model building in relation to the arterial system.

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